TEST 1

1. For the following function:

\[ f(x) = \frac{(4x^2 + 28x + 48)(x - 2)}{(x + 3)(-x^2 + x + 2)} \]

a) (3 points) Find the x and y-intercepts.

b) (2 points) Find the vertical asymptotes, if any.

c) (2 points) Find the horizontal asymptotes, if any.

d) (4 points) List the domain of f(x) in interval notation.

e) (3 points) Sketch f(x) and label all data found in a) thru d).

2. Find the following limits, if they exist and justify your answers.

(5 points each)

a) \( \lim_{u \to -2} \sqrt{(u^4 + 4u - 8)} \)

b) \( \lim_{x \to -3} \frac{4x^3 - 108}{x - 3} \)

c) \( \lim_{t \to -5} \frac{|3x - 15|}{(x - 5)} \)

d) \( \lim_{s \to -1} \frac{3s + 15}{2s^2 + 8s - 10} \)

e) \( \lim_{u \to \infty} \frac{-1}{(2x^2 - 2x - 3)/(1 - 3x - 4x^2)} \)
3. For the following function, find its inverse, sketch and identify both on the same plot. What are their respective domains and ranges? (6 Points)

$$g(x) = (x + 2)^2 \quad x \geq -2$$

4. Solve the following equation for all $$t \in [0, 2\pi)$$ . (6 Points)

$$\cos^2(t) - \frac{3}{4} = 0$$

5. (10 points) What 3 requirements are necessary and sufficient for the $$\lim_{x \to a} f(x)$$ to exist? Sketch any function you can imagine (no equation necessary) so that the limit of f(x) does not exist at 3 points (x=a, x=b and x=c) in its domain ... for a different one of the three reasons at each point.

6. (10 points) List the 3 requirements that are necessary and sufficient for g(x) to be continuous at x=a. Sketch any function you can imagine (no equation necessary) so that g(x) fails continuity at 3 points (x=a, x=b and x=c) in its domain ... for a different one of the three reasons at each point.

7. Sketch the following trigonometric function. On the sketch, identify the coordinates for the y-intercept, at least 3 x-intercepts, at least one maximum and at least one minimum. For full credit you must show the algebra to find each of these answers. (8 Points)

$$f(x) = 2\cos\left(\frac{\pi}{4} x + \frac{\pi}{2}\right)$$
8. Circle either True or False (5 points each)

T  F    If the limit of a function \( f \) exists at \( x=a \), then \( f \) must also exist at \( x=a \). If you choose False, sketch a counter example.

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9. For what values of the constant \( c \) is the function \( g \) continuous at \( x=5 \)? (5 points)

\[
g(x) = \begin{cases} 
  c - x^2 & x \leq 3 \\
  5 + cx & x > 3 
\end{cases}
\]

10. Find the solution(s) to the following logarithmic equation. Remember to show all your algebra. (6 points)

\[
\log(x^2 - 15) - \log x = \log 2
\]

11. Sketch the following function, \( g(x) \). What is it's domain and range? (Hint: What is the domain and range of \( f(x) = e^x \) ? What transformations must \( f(x) \) undergo to become \( g(x) \)?) (5 Points)

\[
g(x) = -e^{x+3} + 2
\]