1. Prove Kepler's third law

2. Find the maximum and minimum values of \( f \) subject to the given constraints:

\[
f(x, y, z) = ye^{x-z}, \quad 9x^2 + 4y^2 + 36z^2 = 36, \quad xy + yz = 1
\]

(You may use a CAS to solve the problem)

3. Evaluate:

\[
\int_{0}^{\sqrt{4-x^2}} \int_{0}^{2\sqrt{4-x^2}} x^2y^2 \, dy \, dx
\]

4. Find the volume of the solid that the cylinder \( r = a \cos \theta \) cuts out of the sphere of radius \( a \) centered at the origin.

5. Show that

\[
\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{1}{\sqrt{x^2 + y^2 + z^2}} e^{-(x^2 + y^2 + z^2)} \, dx \, dy \, dz = 2\pi
\]

6. Use Maple or Mathematica to plot the vector field

\[
\mathbf{F}(x, y) = \langle y^2 - 2xy, 3xy - 6x^2 \rangle
\]

Explain the appearance by finding the set of points \((x, y)\) such that \( \mathbf{F}(x, y) = \mathbf{0} \).

7. Use Stoke's theorem to evaluate \( \iint_S \text{curl} \mathbf{F} \cdot d\mathbf{S} \) where

\[
\mathbf{F}(x, y, z) = \langle -yz, xz, 3(x^2 + y^2)z \rangle \quad \text{and} \quad S \text{ is the part of the paraboloid } z = x^2 + y^2 \text{ that lies inside the cylinder } x^2 + y^2 = 1, \text{ oriented upward.}
\]

8. Use the divergence theorem to find the outward flux of the vector field

\[
\mathbf{F}(x, y, z) = \langle x^2, y^2, 4z^2 \rangle \quad \text{across the boundary of the rectangular prism: } 0 \leq x \leq 1, \quad 0 \leq y \leq 5, \quad 0 \leq z \leq 5
\]
9. For a surface \( z = f(x, y) \), recall that a normal vector to the tangent plane at \((a, b, f(a, b))\) is \( \langle f_x(a, b), f_y(a, b), -1 \rangle \). Show that the surface area formula can be rewritten as

\[
\text{Surface area} = \iint |\mathbf{n}| \quad dA
\]

where \( \mathbf{n} \) is the unit normal vector to the surface. Use this formula to set up a double integral for the surface area of the top half of the sphere \( x^2 + y^2 + z^2 = 4 \) (Hint: Use the gradient to compute the normal vector and substitute \( z = \sqrt{4-x^2-y^2} \) to write the integral in terms of \( x \) and \( y \).) For a surface such as \( y = 4 - x^2 - z^2 \), it is convenient to think of \( y \) as the dependent variable and double integrate with respect to \( x \) and \( z \). Write out the surface area formula in terms of the normal vector for this orientation and use it to compute the surface area of the portion of \( y = 4 - x^2 - z^2 \) inside \( x^2 + z^2 = 1 \) and to the right of the \( xz \)-plane.